Shape Optimization Using Reflection Lines

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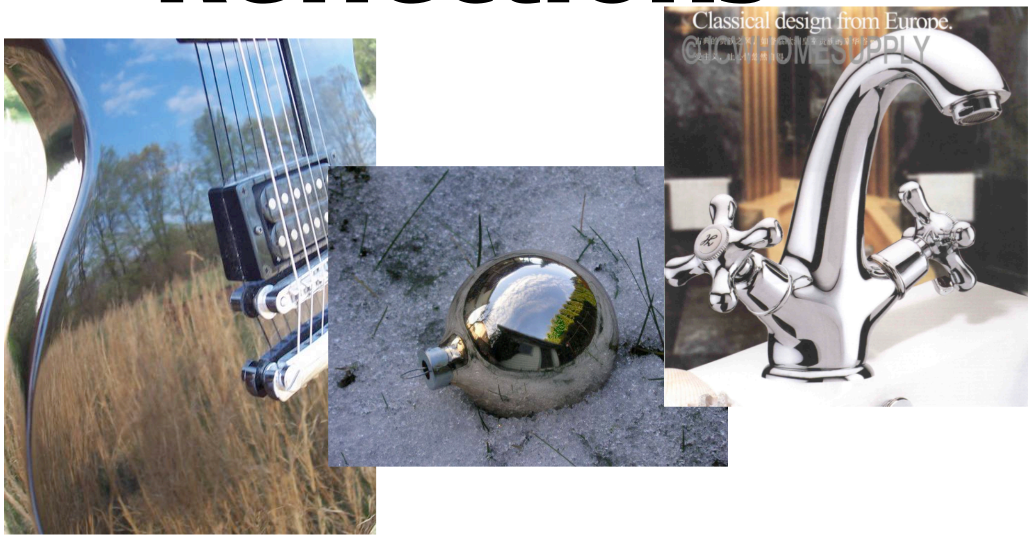
Reflections

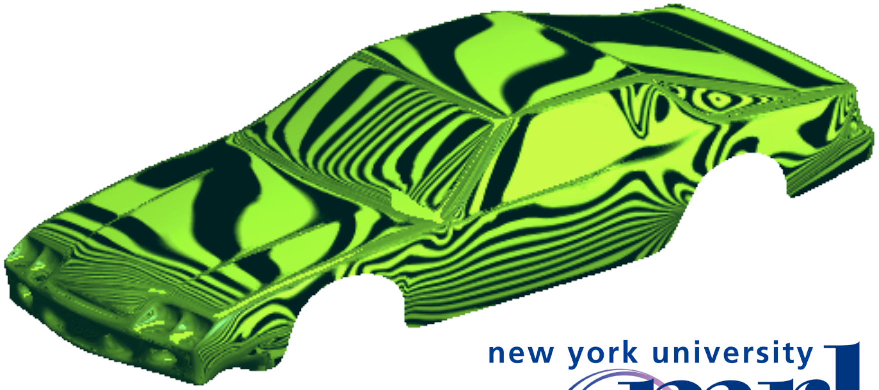


- are sensitive to surface shape
- depend on local quantities
- depend on viewer location

"Cloud Gate"
Anish Kapoor

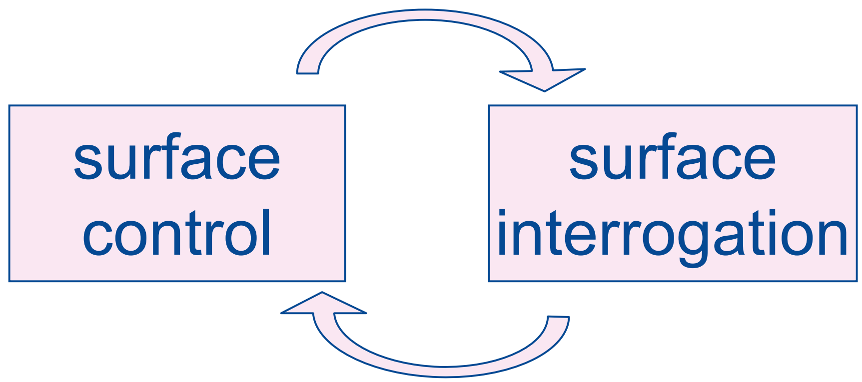
Reflection Lines

- Capture aspects of general reflections
- Show surface imperfections better than lighting only
- Tool for surface quality assessment
- Interactive rendering, easy to implement



Problem

- Surface quality and shape design complimentary
- Control of shape has indirect effect on quality



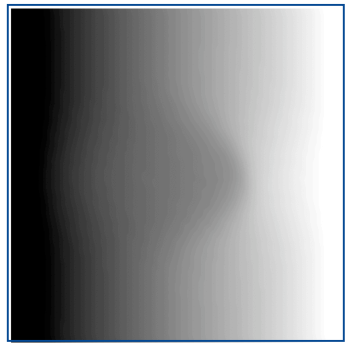
Formulate surface editing as an optimization problem

Problem

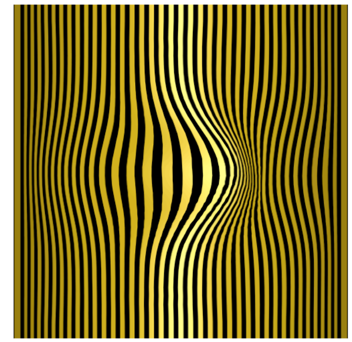
Surface Interrogation

Surface *f*

Reflection Function *θ(f)*



Reflection Lines

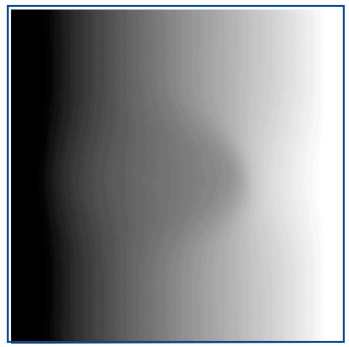


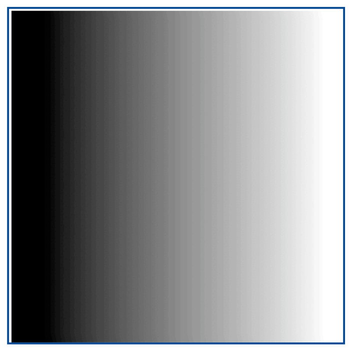
Problem

Surface Interrogation

Surface *f*

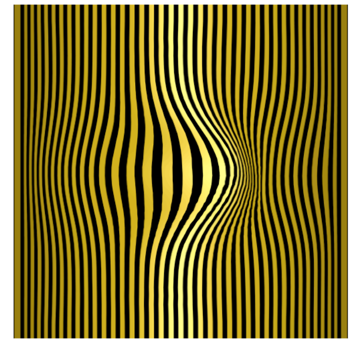
Reflection Function *θ(f)*

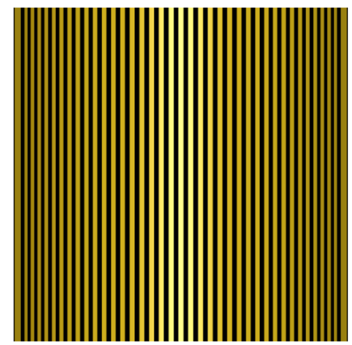




User defined
Reflection Function *θ**

Reflection Lines





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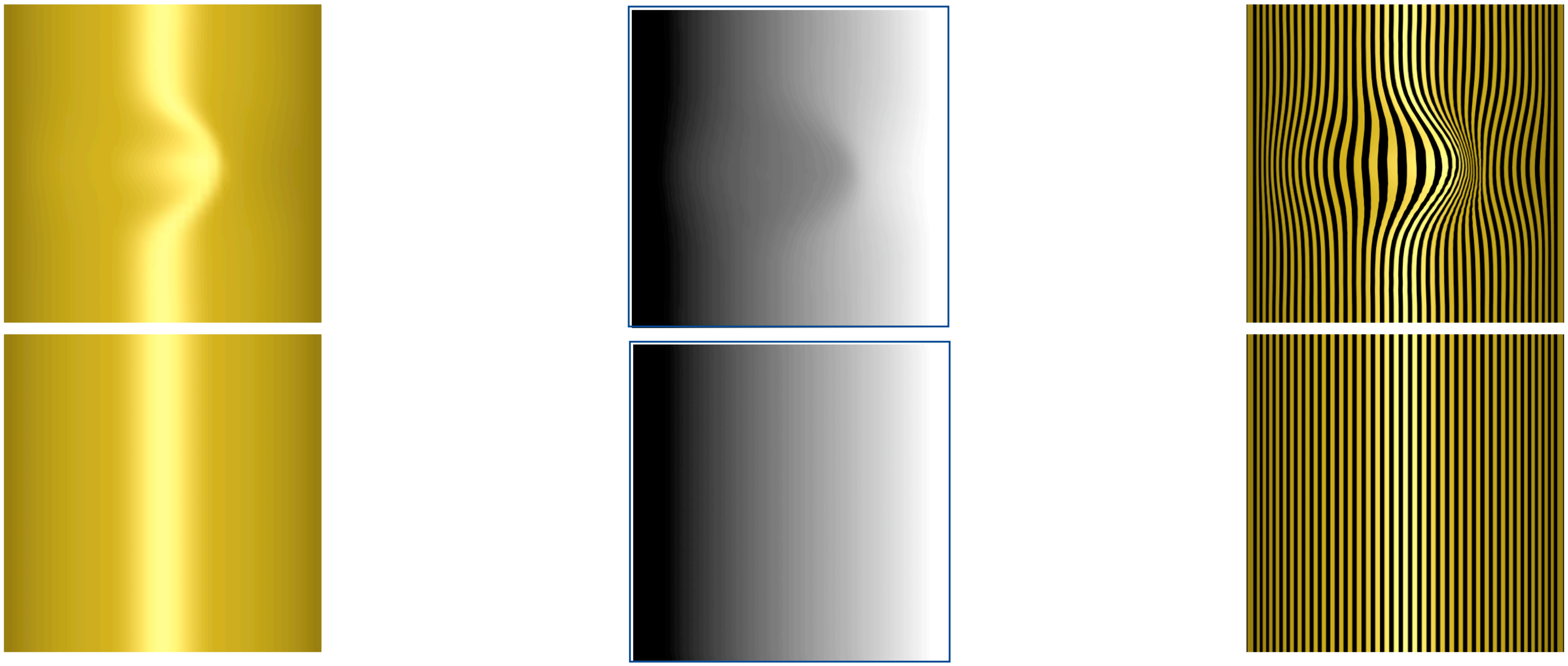
Problem

Surface Interrogation

Surface *f* | Reflection Function *θ(f)* | Reflection Lines



User-defined
Reflection Function *θ**

$$\int (\theta(f) - \theta^*)^2 \rightarrow \min$$

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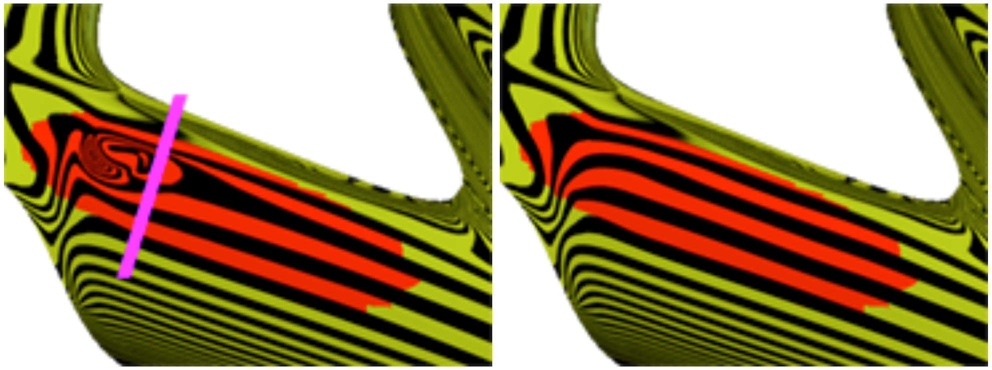
Our Solution

- Interactive surface modeling tool based on reflection line optimization
- Mesh based - discretization of reflection lines
- Smoothing, warping, changing line density and direction, image based reflection



before after

Approach

- Local parameterization over image plane
- Triangle-based discretization of derivatives

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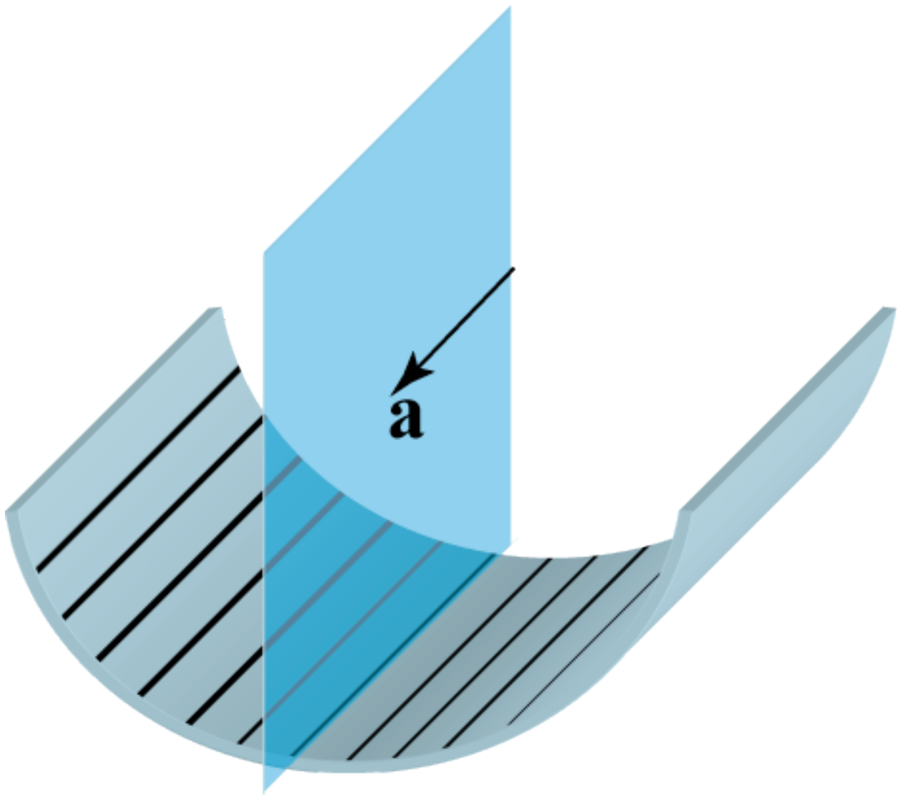
Related Work

- Klass 1980
  - differential-geometric description
- Horn 1986
  - shape from shading
- Loos, Greiner and Seidel 1999
  - reflection lines on NURBS
- Hildebrandt, Polthier and Wardetzky 2005
- Grinspun, Gingold, Reisman and Zorin 2006
  - discrete shape operators

Reflection Line Function



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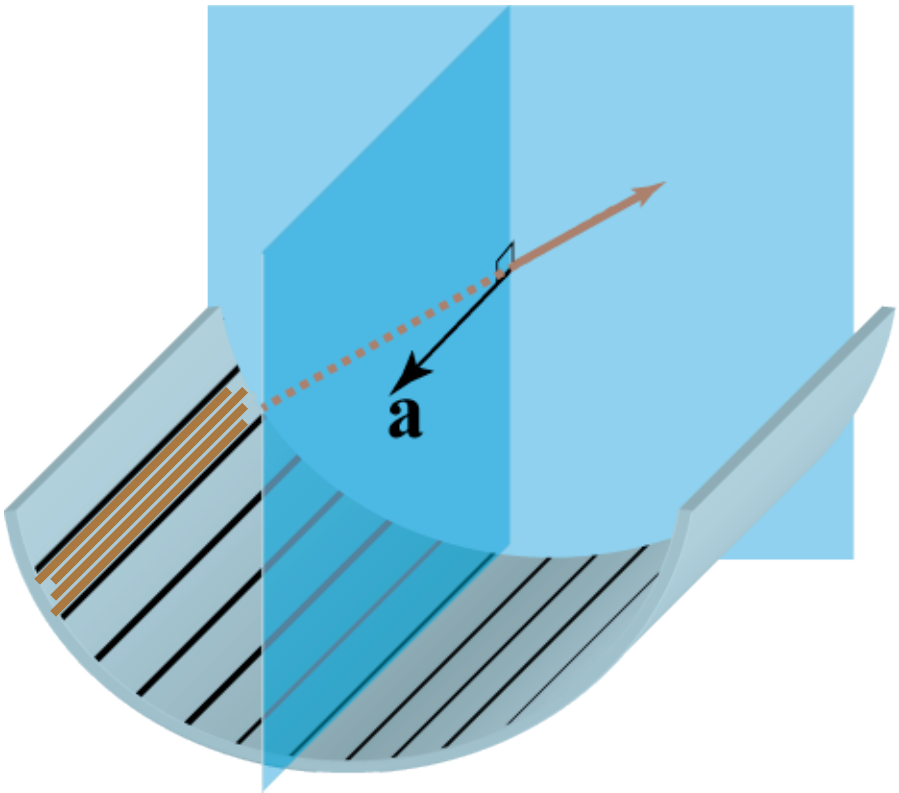
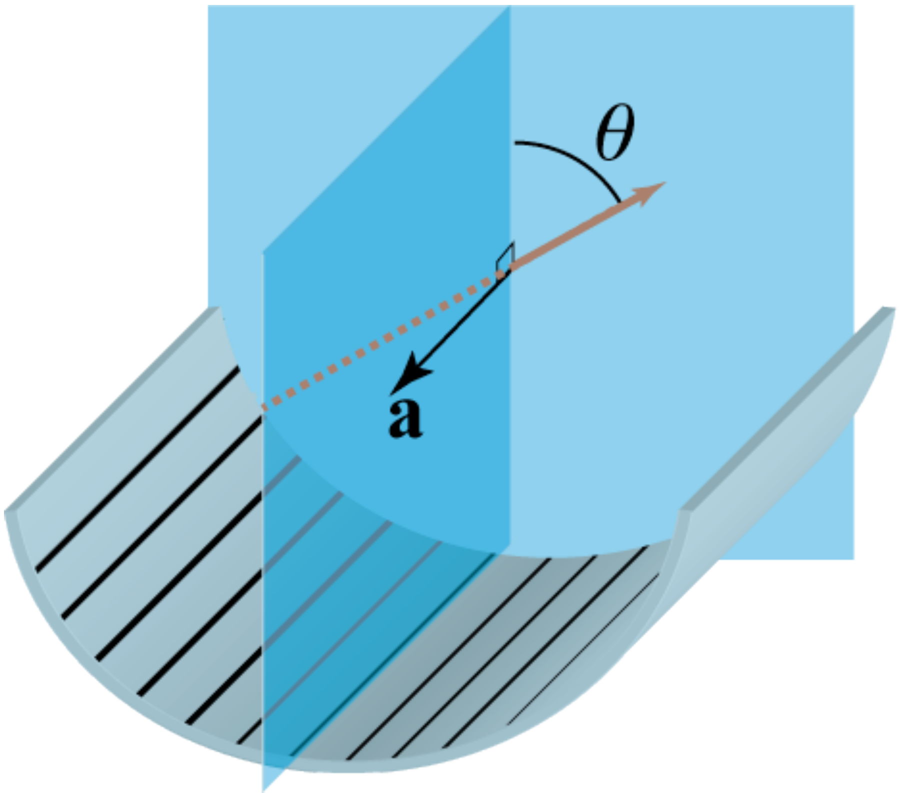
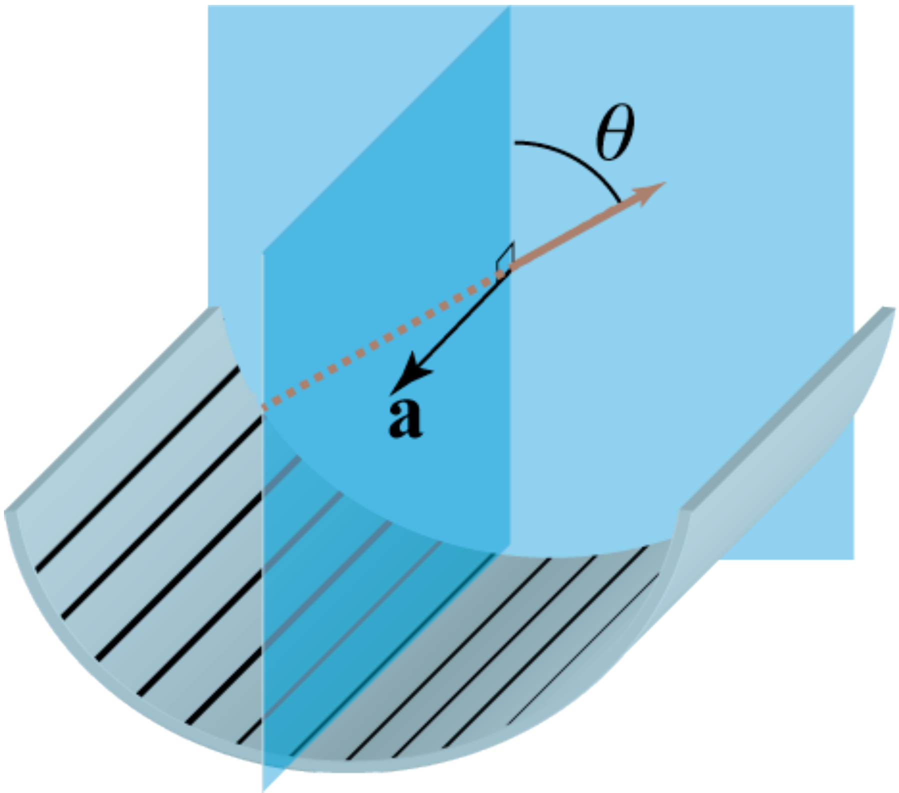
Reflection Line Function

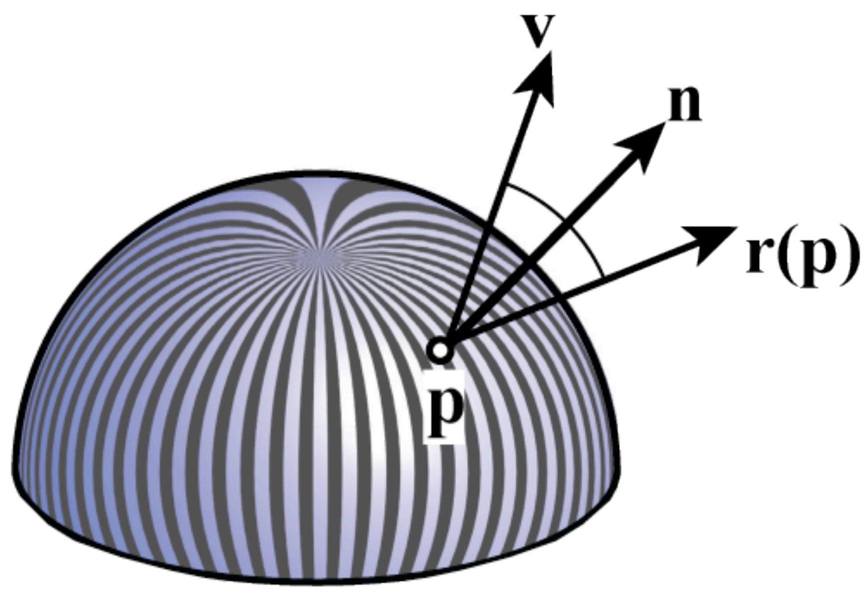


Reflection Line Function

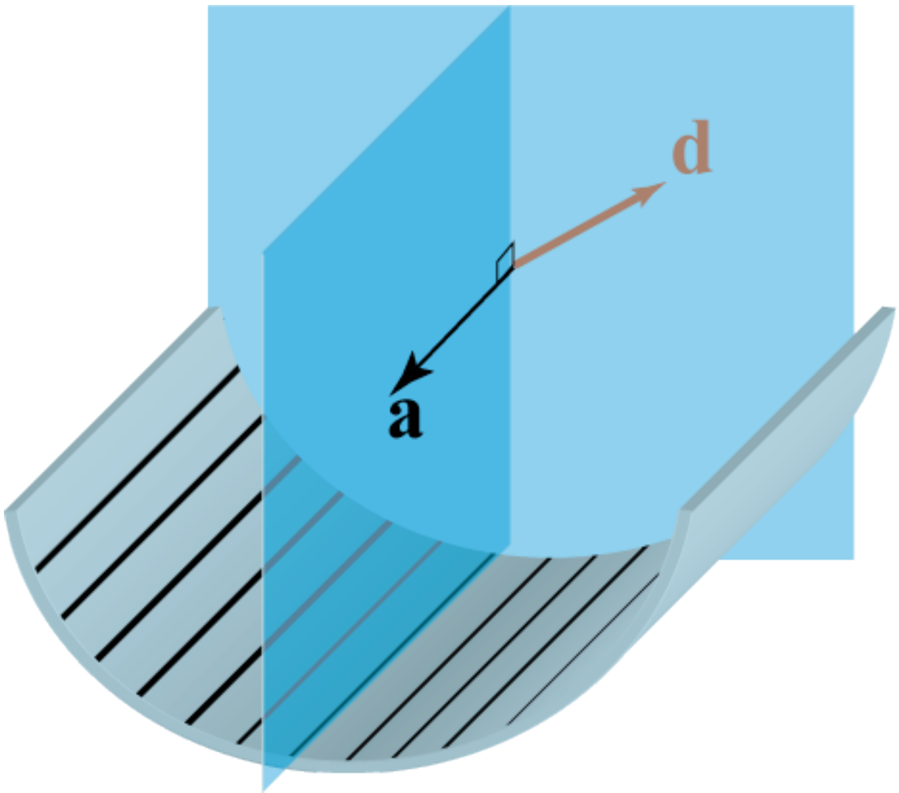


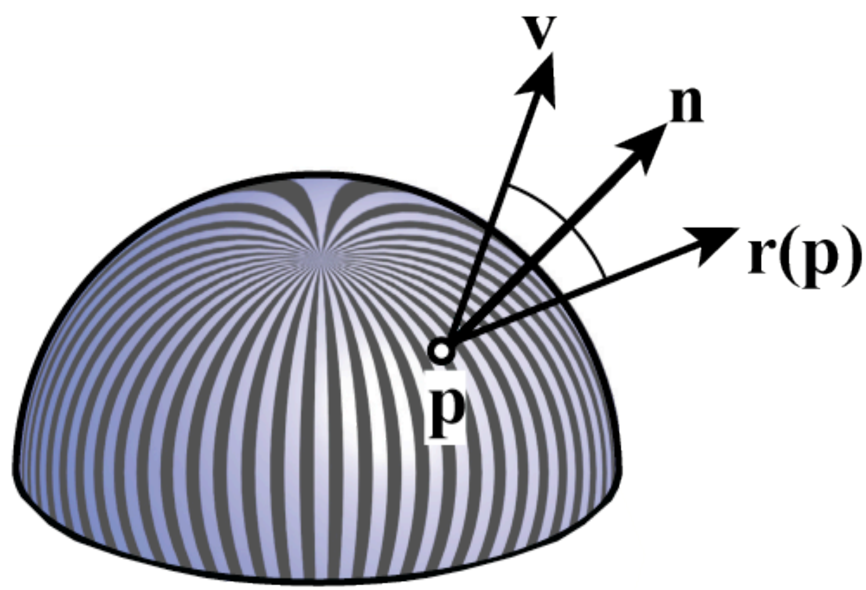
Reflection Line Function



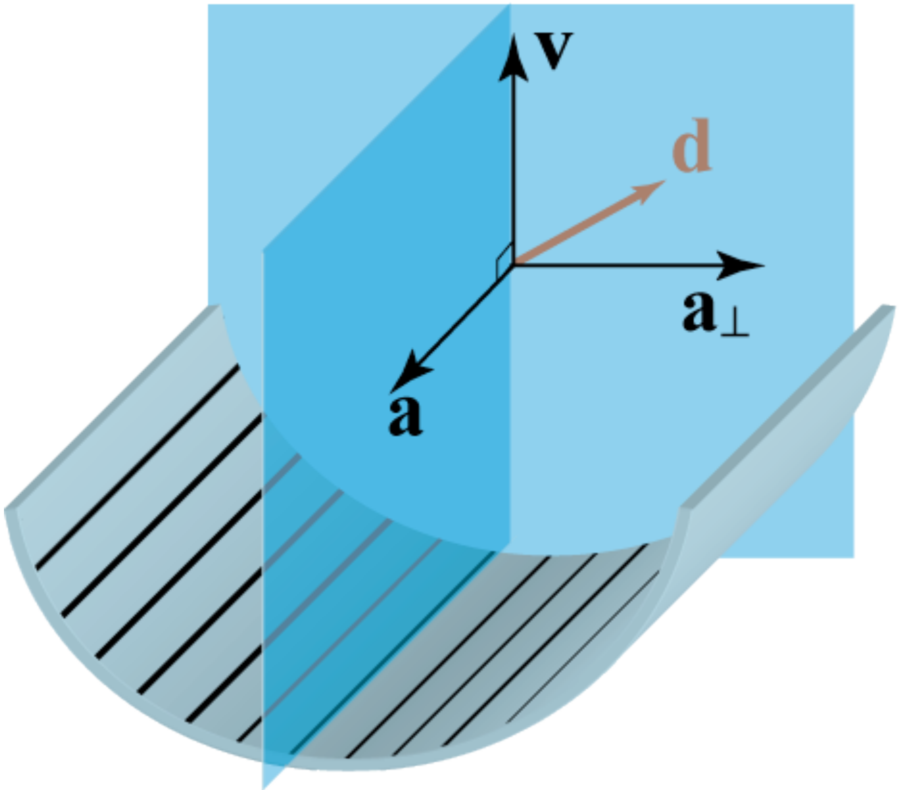


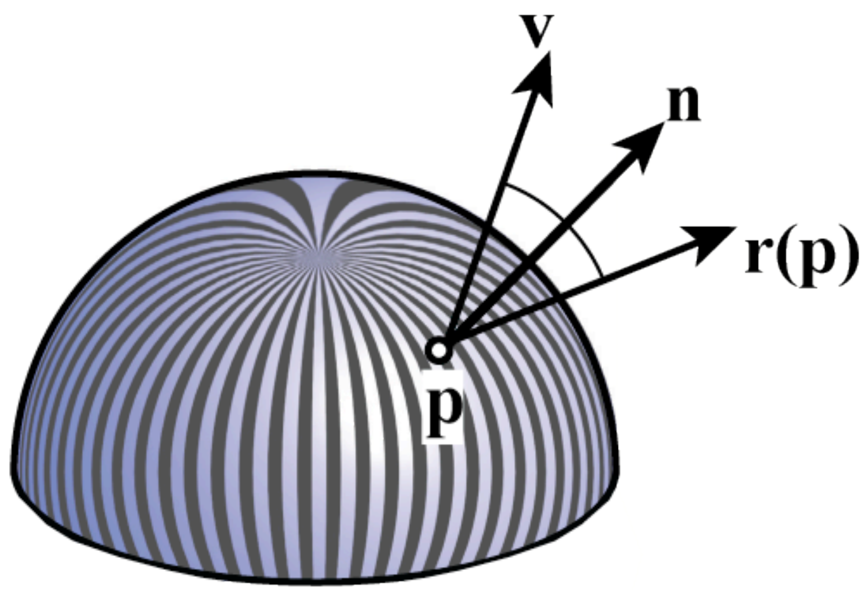
Reflection Line Function



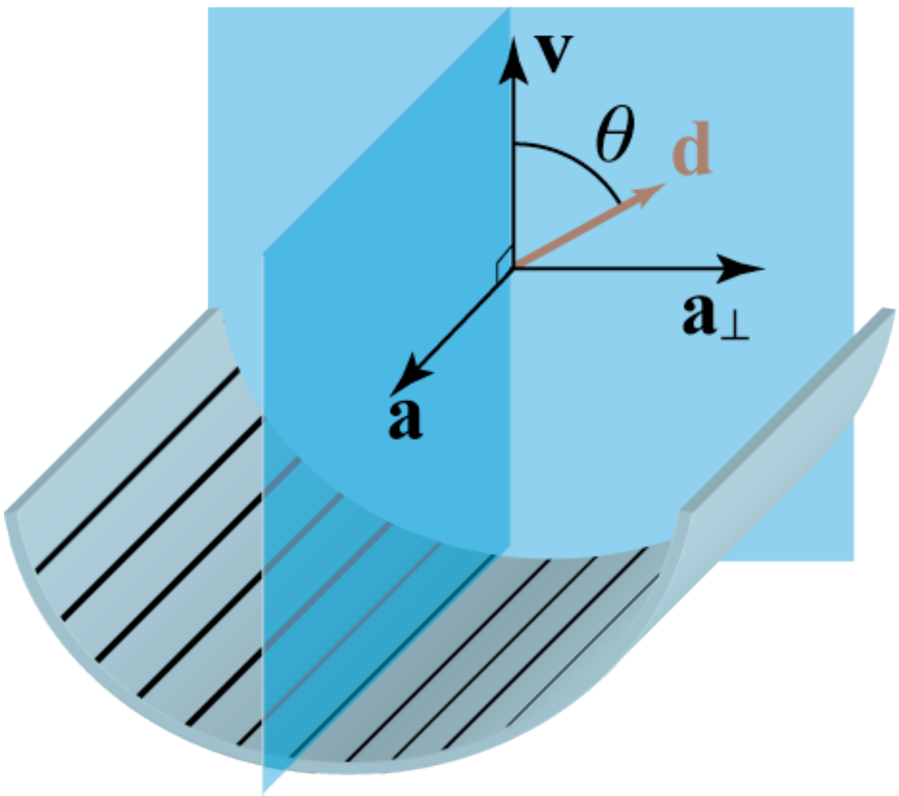


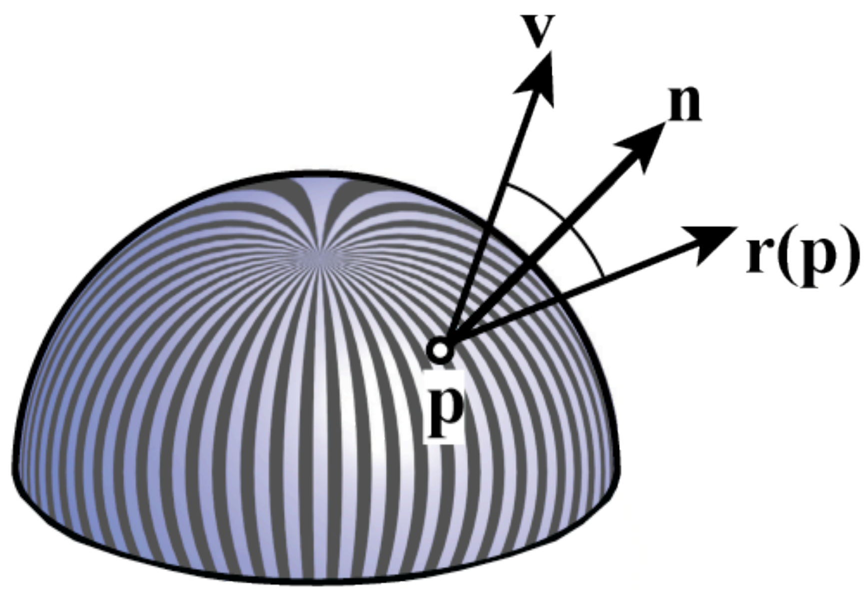
Reflection Line Function





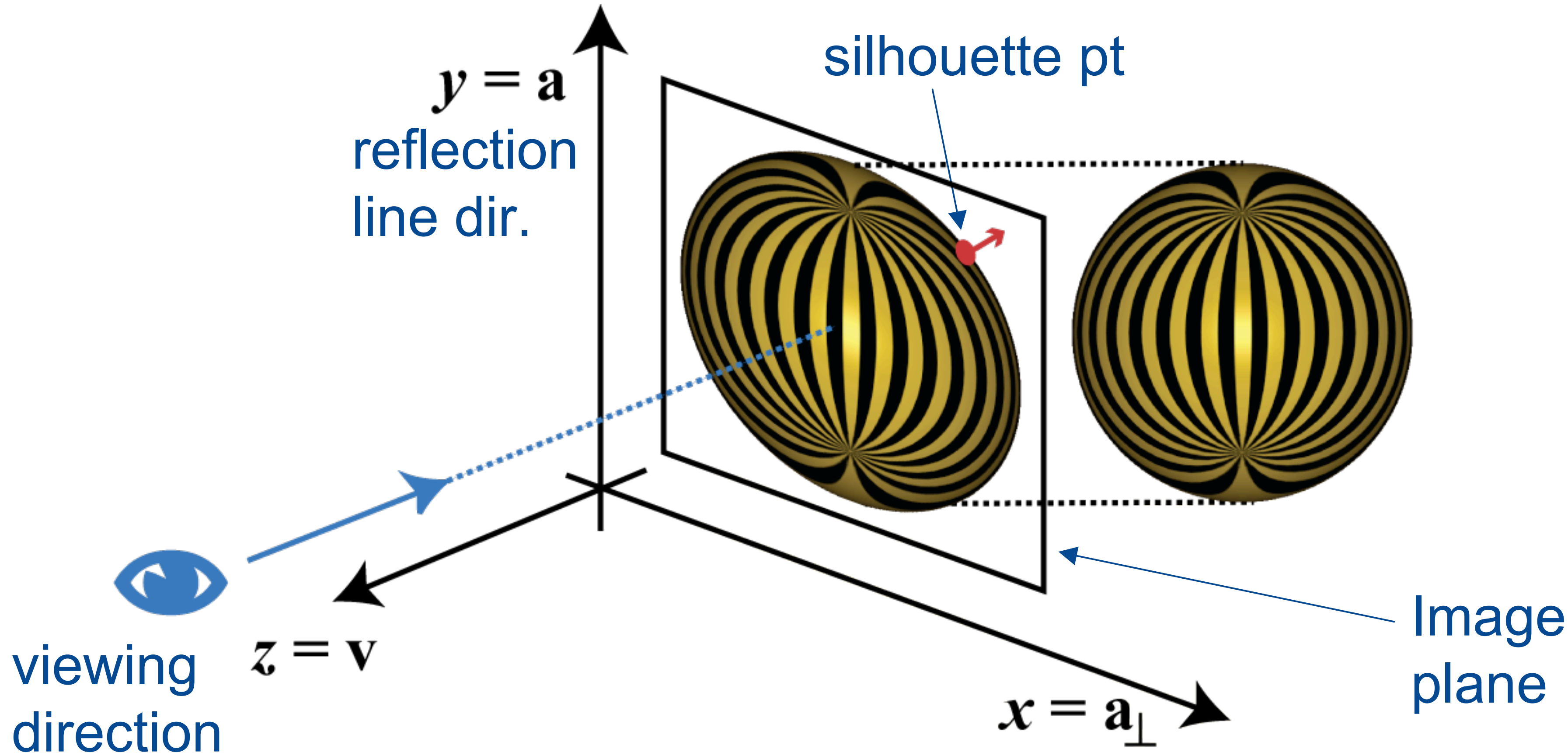
Reflection Line Function





$$\theta(\mathbf{p}) = \arctan((\mathbf{r}(\mathbf{p}) \cdot \mathbf{a}_\perp), (\mathbf{r}(\mathbf{p}) \cdot \mathbf{v}))$$

Image-Plane Parameterization



surface as height field

$$\theta(x, y) = \arctan(2 f_x, (1 - f_x^2 - f_y^2))$$

Reflection Functionals

Function-based

$$\int_S (\cos\theta - \cos\theta^*)^2 + (\sin\theta - \sin\theta^*)^2 dxdy \rightarrow \min$$

$$\theta|_{\partial S} = \theta_0$$

User defined reflection func.

Gradient-based

$$\int_S (\nabla\theta - \nabla\theta^*)^2 dxdy \rightarrow \min$$

$$\theta|_{\partial S} = \theta_0 \, , \frac{\partial}{\partial n}\theta|_{\partial S} = \varphi_0$$

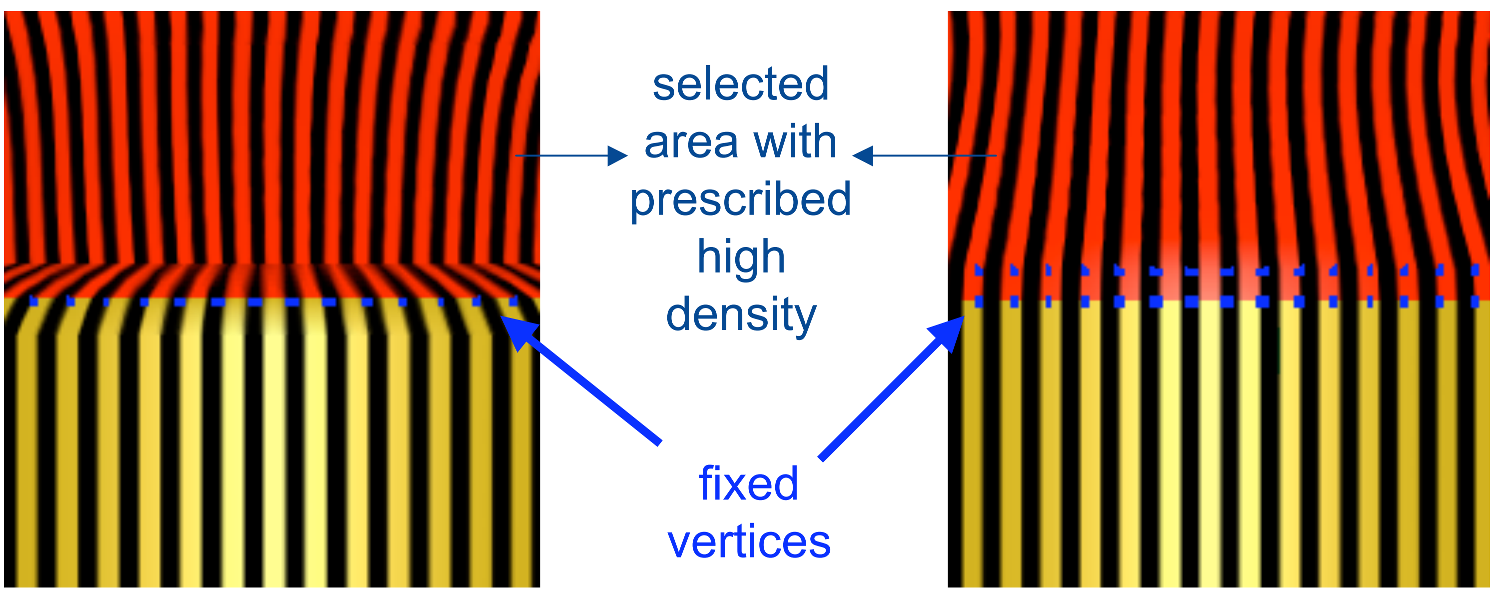
Reflection Functionals

Function-based

- Euler-Lagrange 2nd order
- Can prescribe only function values on boundary
- No blending with rest of surface

Gradient-based

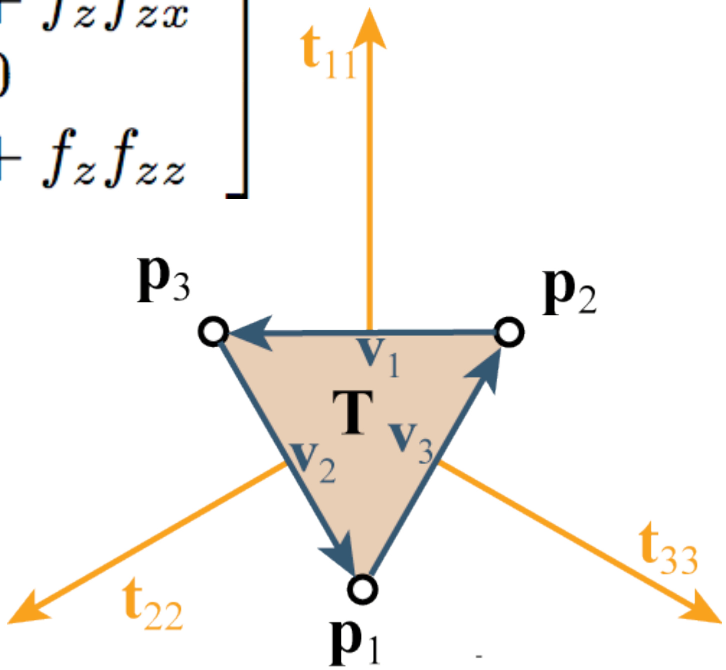
- Euler-Lagrange 4th order
- Can prescribe function and derivative values on boundary
- Smooth blending at selection boundaries



Gradient Discretization

$$\nabla\theta = \frac{a\nabla b - b\nabla a}{a^2+b^2} \qquad a = 2f_x\ ,\ b = 1 - f_x^2 - f_z^2$$

$$\nabla a = 2\begin{bmatrix} f_{xx} \\ 0 \\ f_{xz} \end{bmatrix},\ \nabla b = -2\begin{bmatrix} f_x f_{xx} + f_z f_{zx} \\ 0 \\ f_x f_{xz} + f_z f_{zz} \end{bmatrix}$$



Triangle-centered

Piecewise linear finite elements

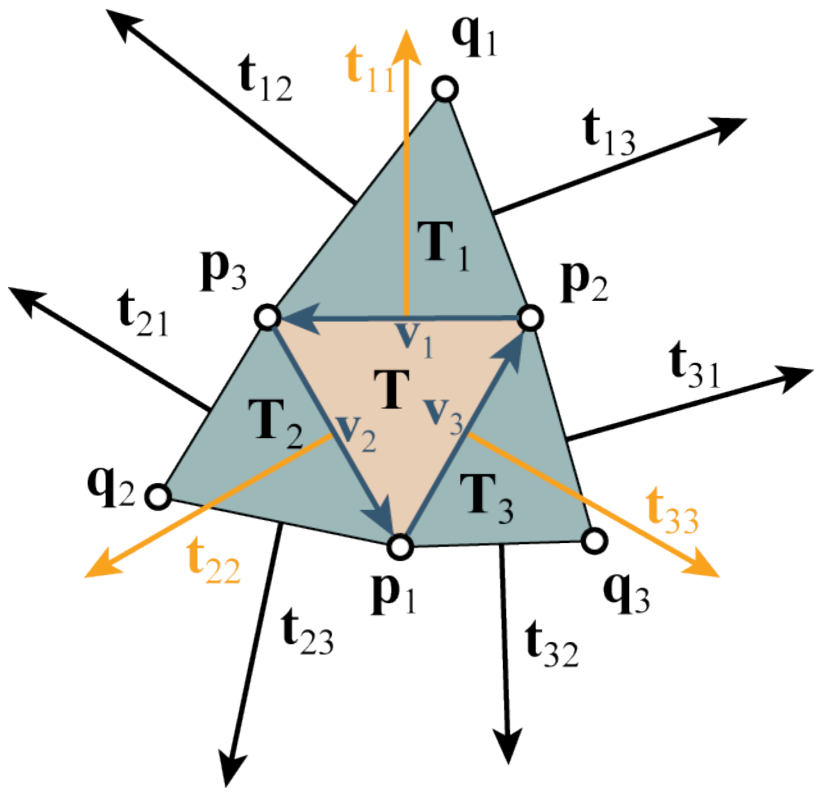
$$\nabla_{discr} f_T = \frac{1}{2A}\sum_{i=1,2,3} f(\mathbf{p}_i)\mathbf{t}_{ii}$$

Hessian Discretization

At least 6 DOF per stencil needed -- triangle with flaps

<u>Triangle-averaged</u>

Averaging shape operators over triangle edges [Hildebrandt et al. 2005], [Grinspun et al. 2006]



Hessian Discretization

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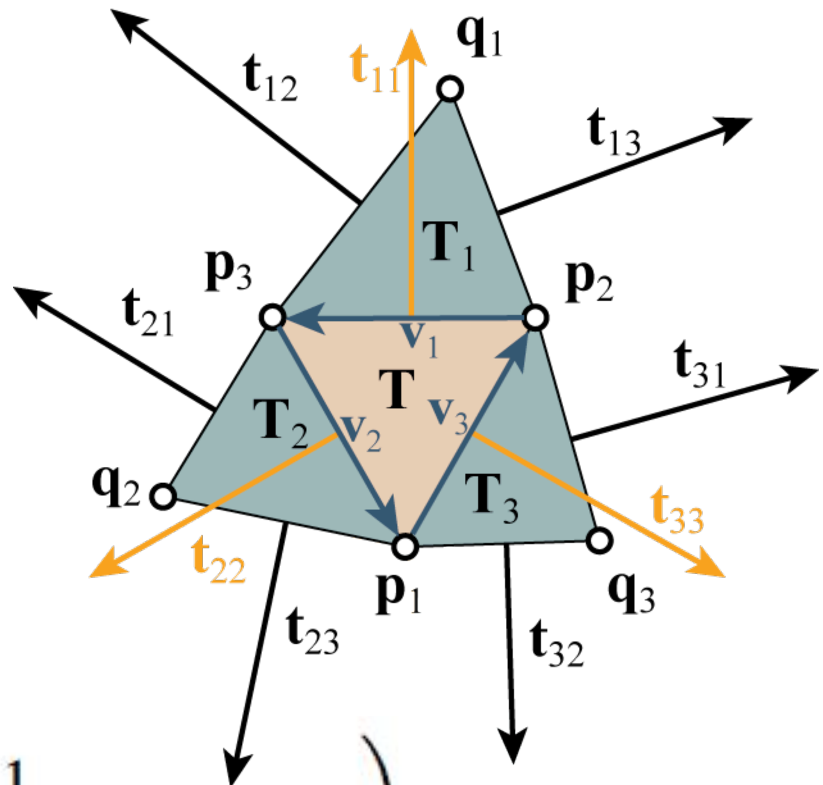


<u>Triangle-averaged</u>

Averaging shape operators over triangle edges [Hildebrandt et al. 2005], [Grinspun et al. 2006]

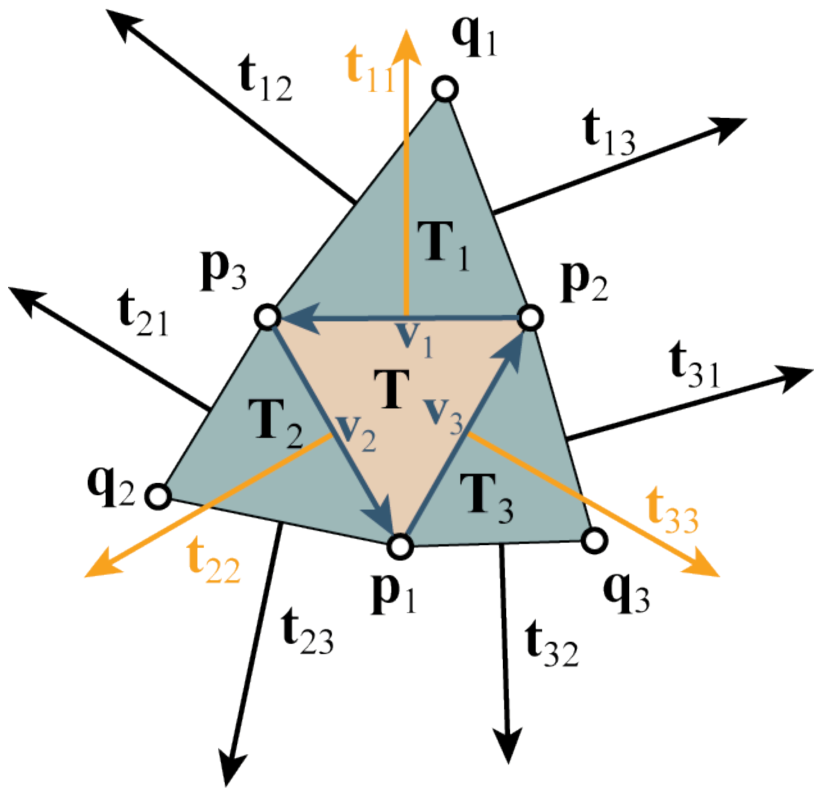
A, A_i : area factors

$$H(f) = \frac{1}{A}\left(\sum_{i,j,j\neq i}\frac{1}{A_j} f(\mathbf{q}_j)\mathbf{t}_{ii}\otimes\mathbf{t}_{ij} + \sum_{i}\frac{1}{A_i} f(\mathbf{p}_i)\mathbf{t}_{ii}\otimes\mathbf{t}_{ii}\right)$$

Hessian Discretization

Triangle-averaged

- Pros
  - robust
  - simple
  - consistent for special meshes.
- Cons
  - for general meshes, mesh-dependent error



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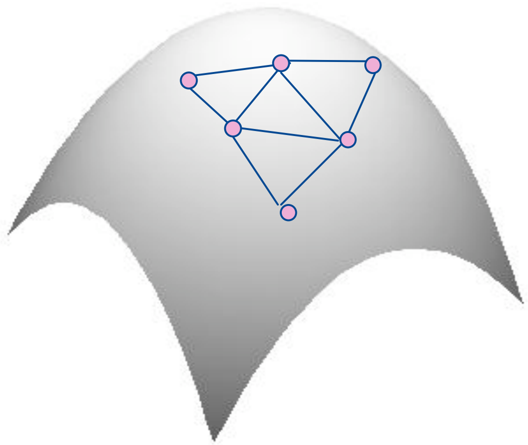
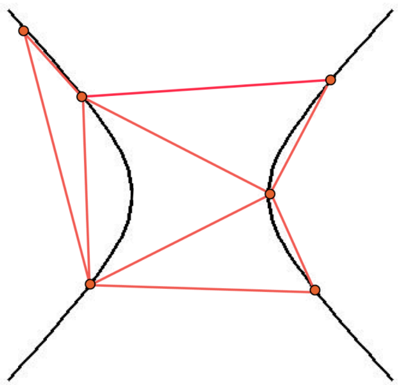
Hessian Discretization

Quadratic interpolation

- Unique quadratic function to interpolate vertices of stencil
- Use quadratic term coefficients



- Pros
  - Consistent
  - Less dependent on mesh connectivity
- Cons
  - Less robust - if vertices on or close to a conic no solution or large coefficients



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Hessian Discretization

Hybrid discretization

- Use triangle-averaged scheme when quadratic interpolation unstable
- Evaluate stability by comparing coeffs to $1/l_{max}^2$

- Pros:
  - More robust
  - More accurate
- Cons:
  - Large errors for some meshes

Hessian Discretization

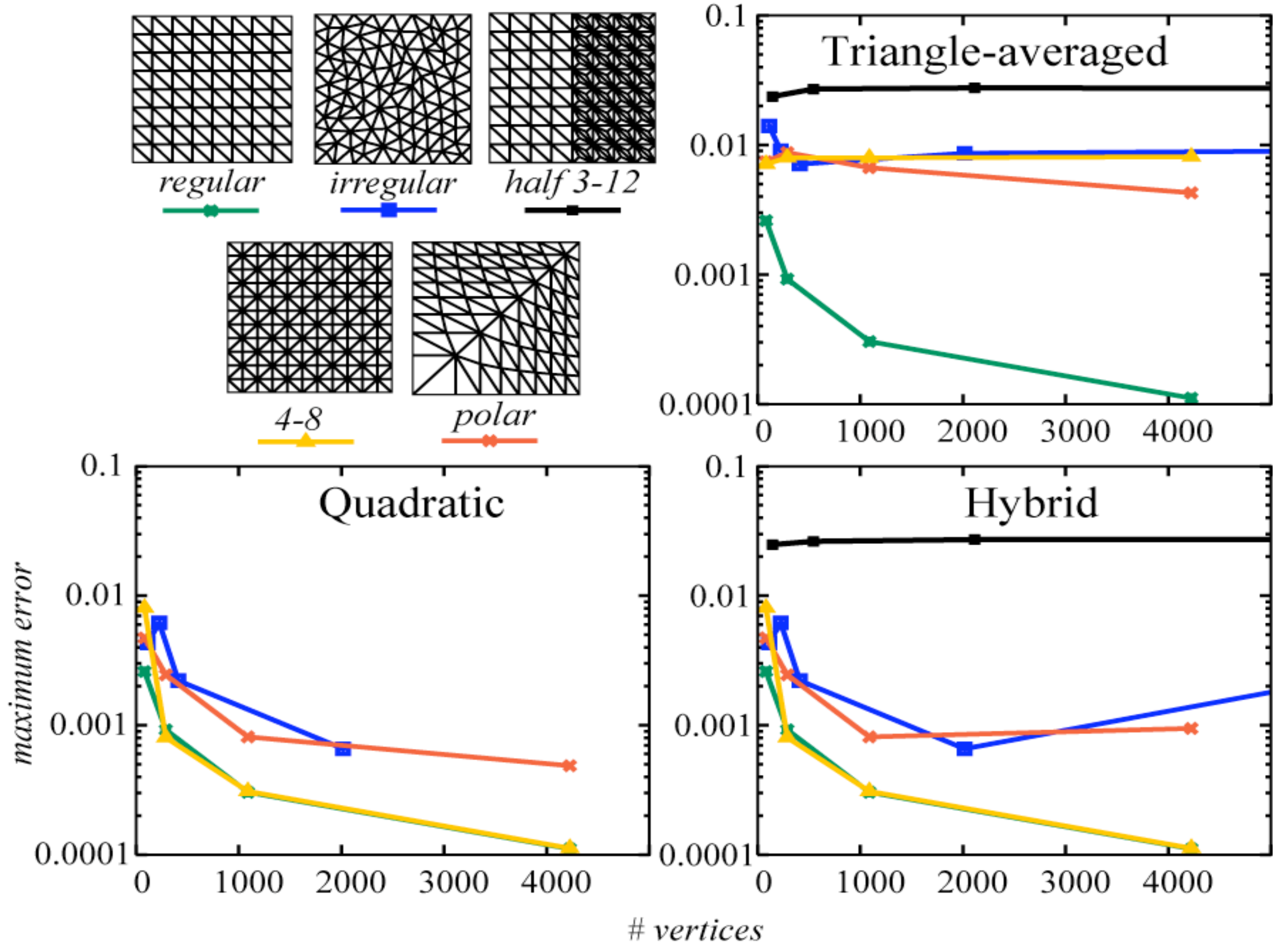
Initial

Quad fit

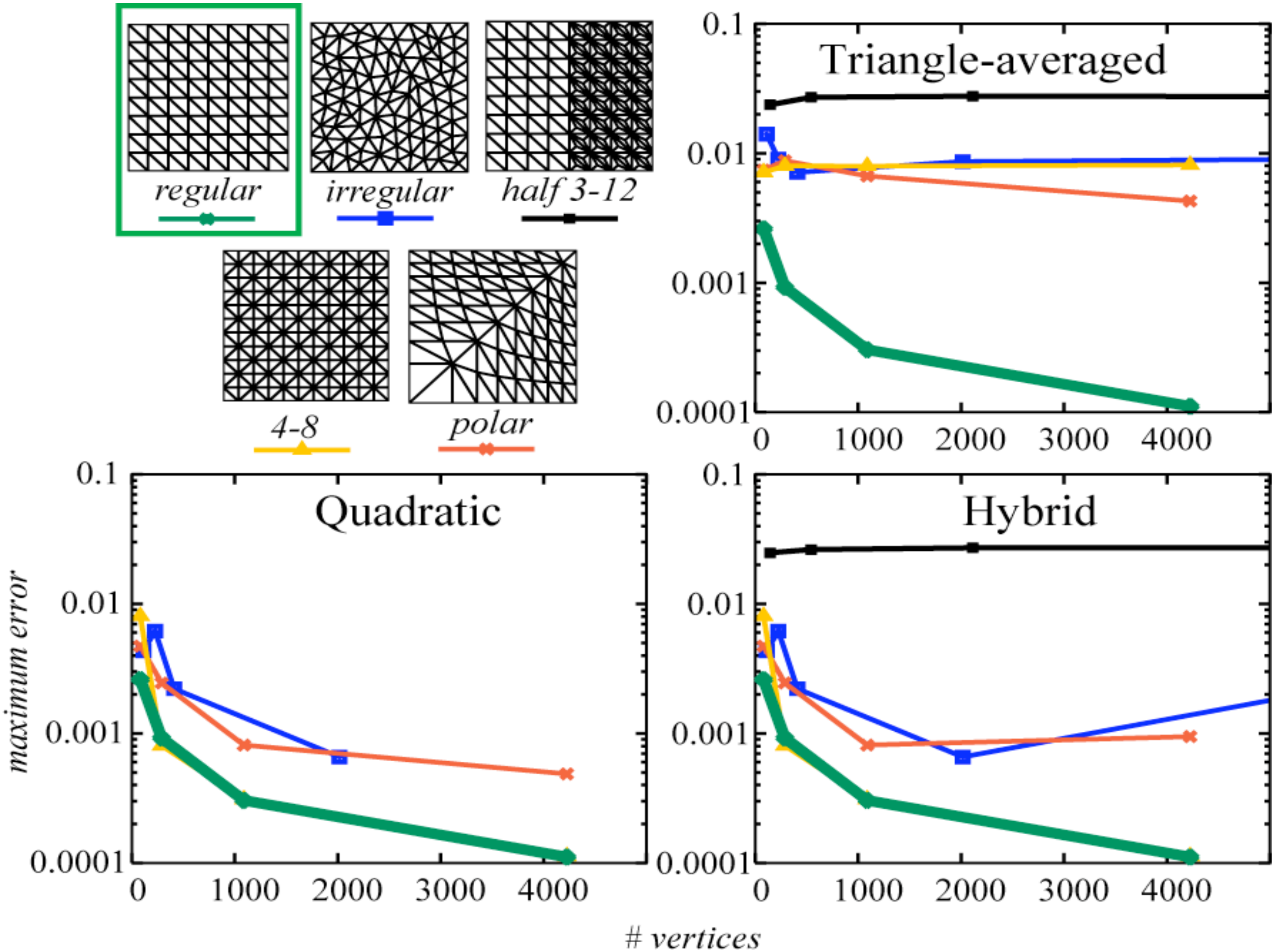
Tri-avg

Hybrid

Hessian Discretization



Hessian Discretization



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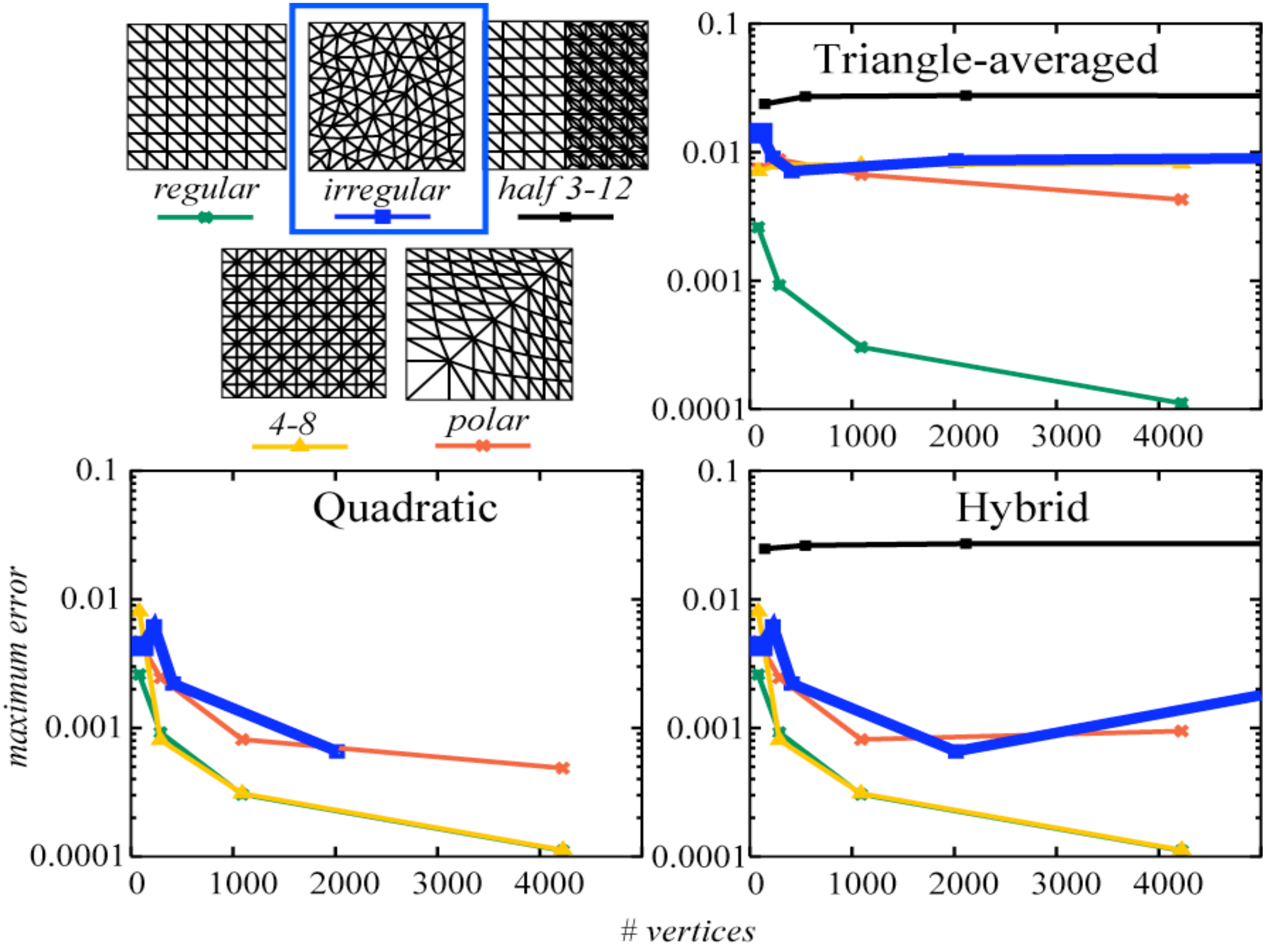
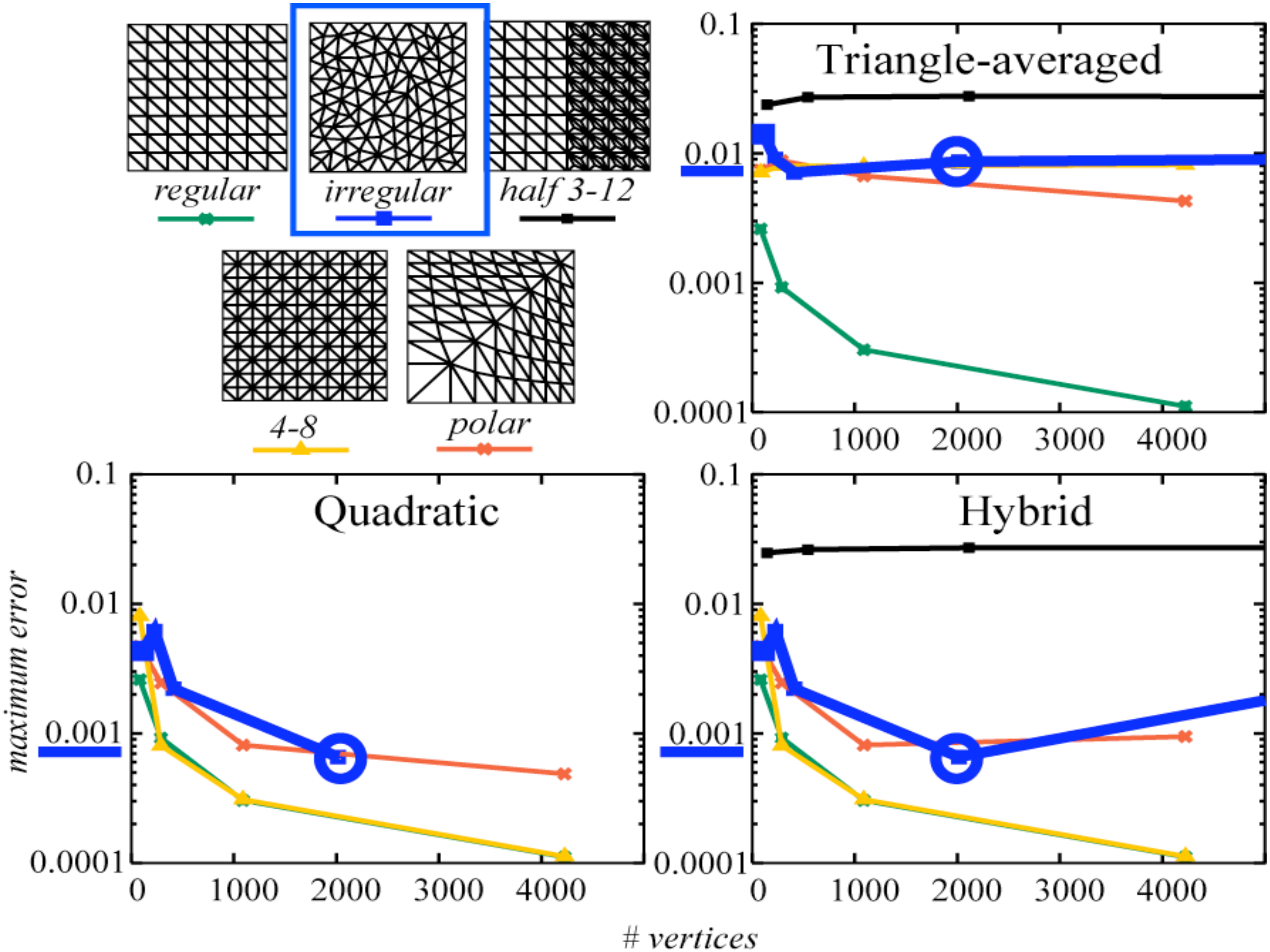


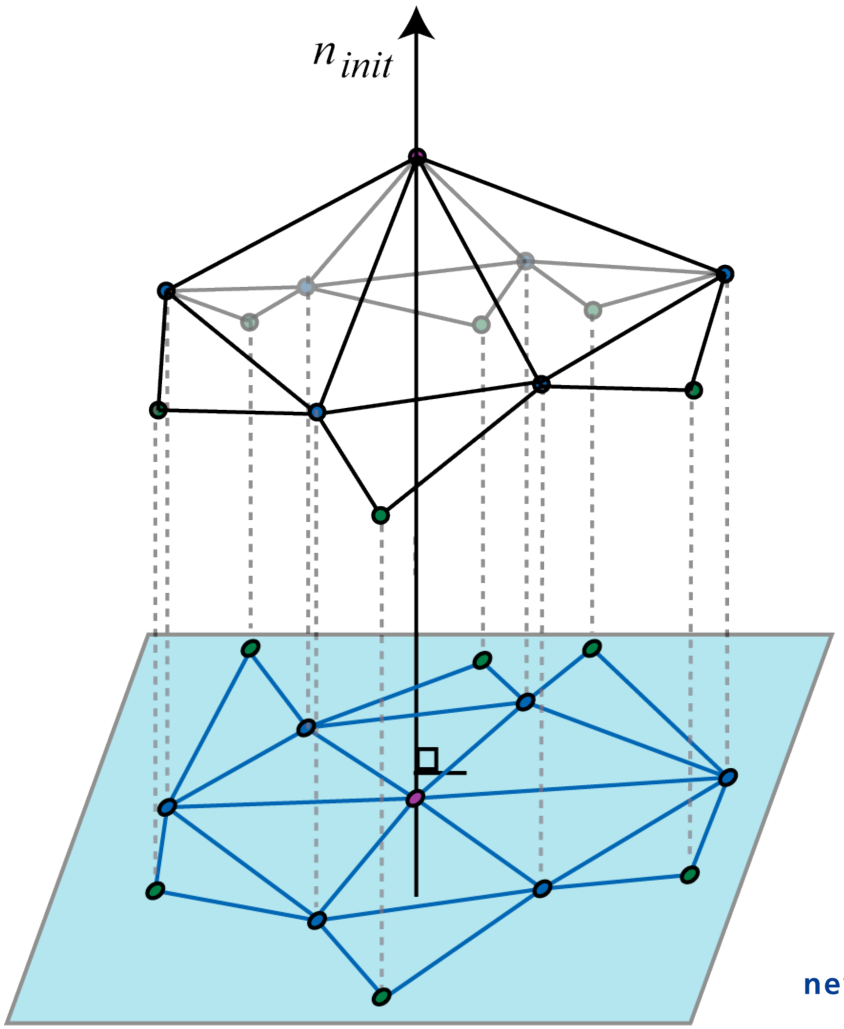
Hessian Discretization



Normal Estimation

Local quadratic fit ($O(h^2)$)

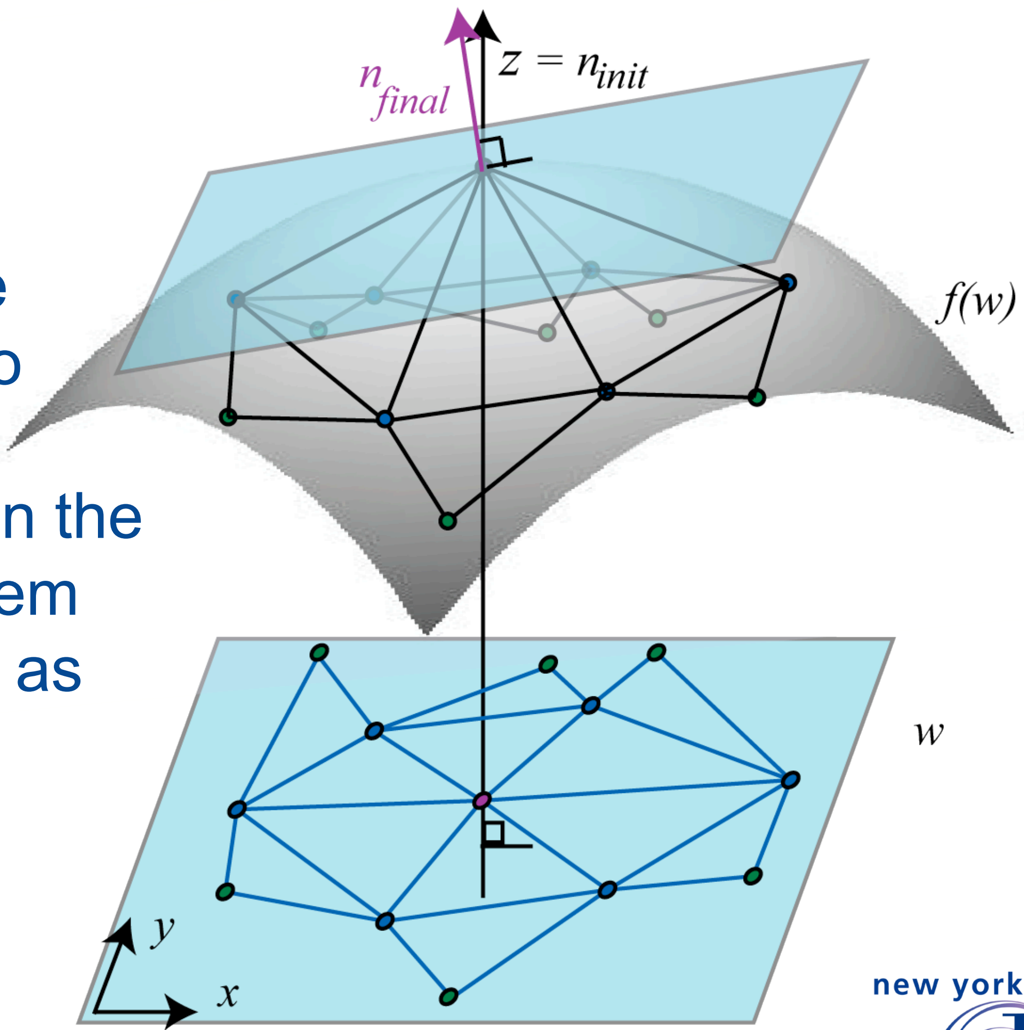
1. Project to plane perpendicular to initial normal



Normal Estimation

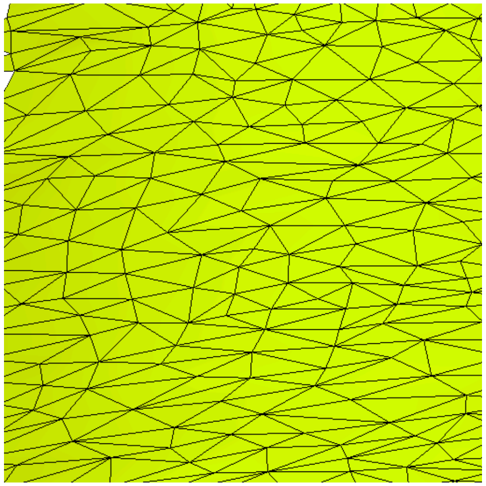
Local quadratic fit
($O(h^2)$)

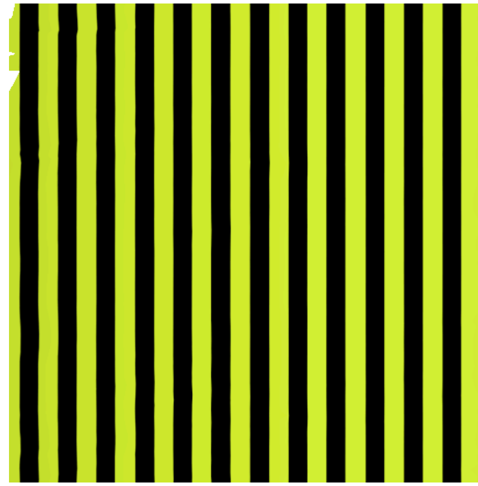
1. Project to plane perpendicular to initial normal
2. Fit a quadratic in the new coord system
3. Use the normal as vertex normal



Normal Estimation

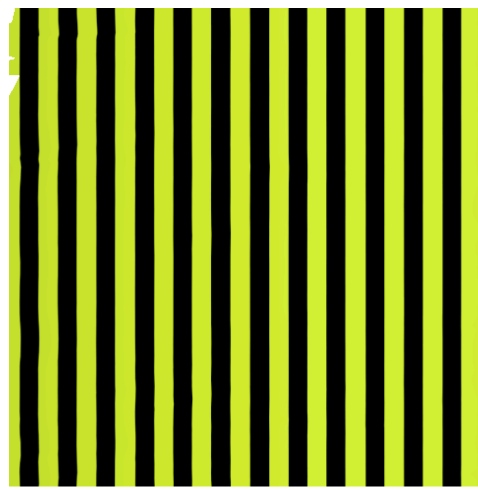
mesh





analytic normals

averaged face normals



quadratic fit normals

Interactive Speeds

- Linearizing the energy does not work
- Full non-linear Newton or gradient-only methods too expensive

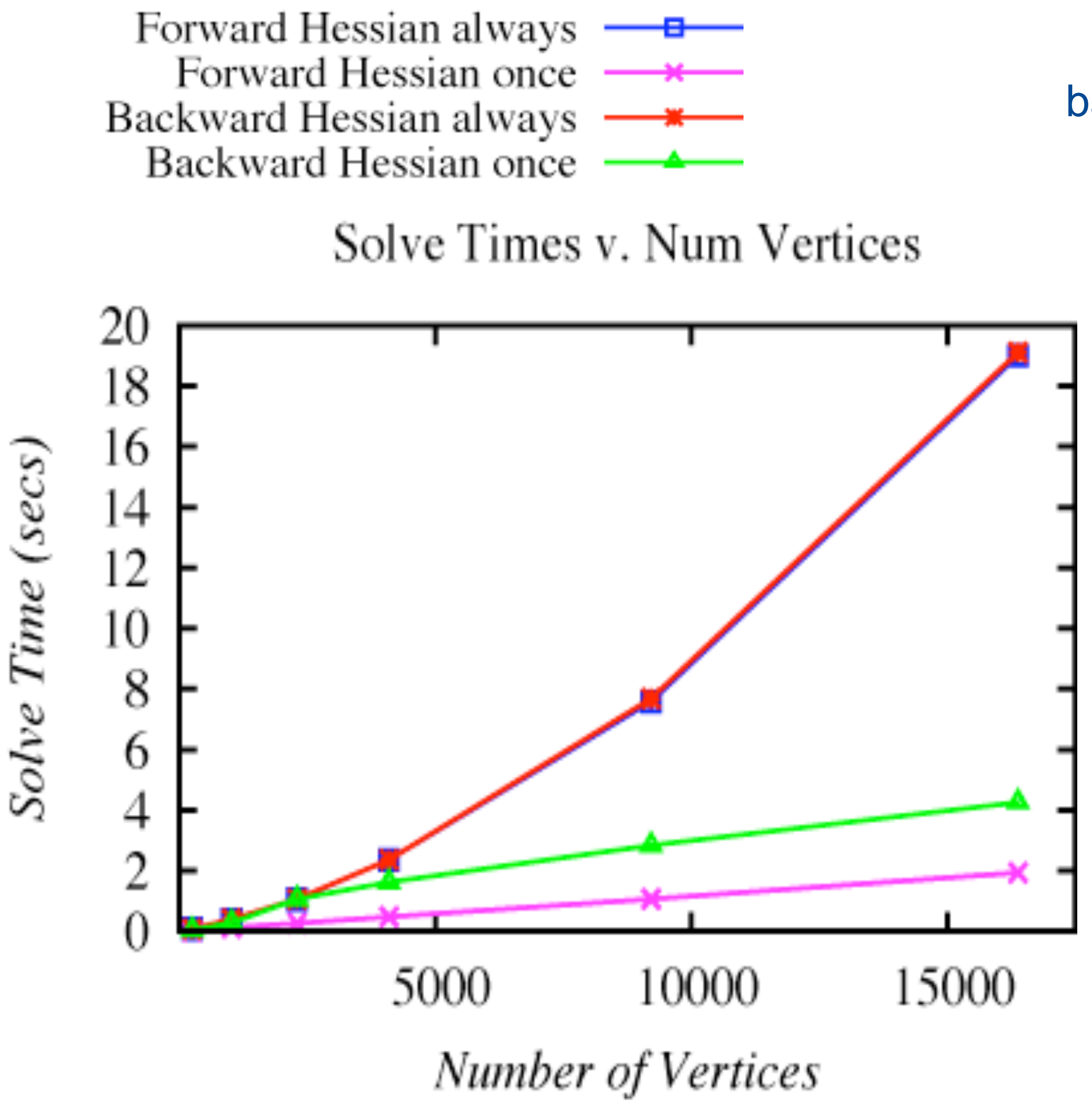
Solution:

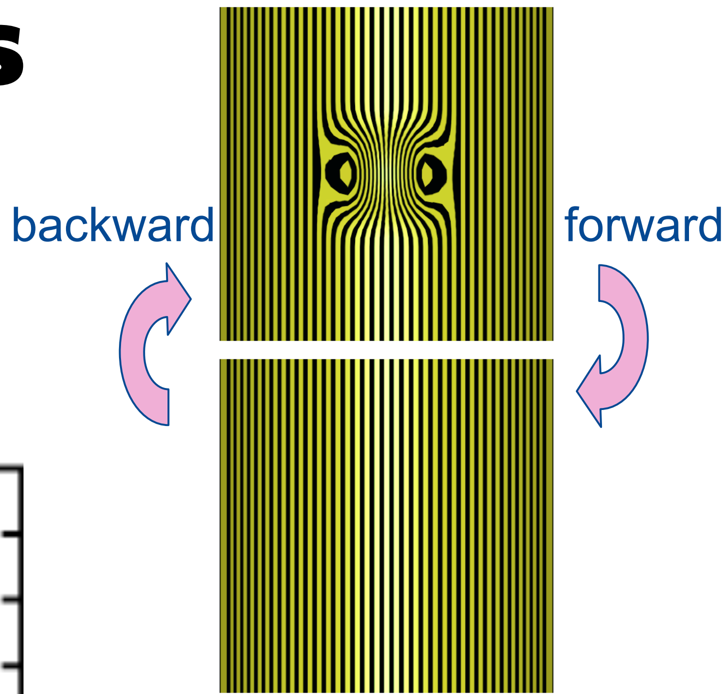
Inexact Newton method with line search

- Compute and factor Hessian once and reuse
- Compute Hessian for the linearized problem

$$\theta_{lin}(x, y) = 2f_x$$

Interactive Speeds



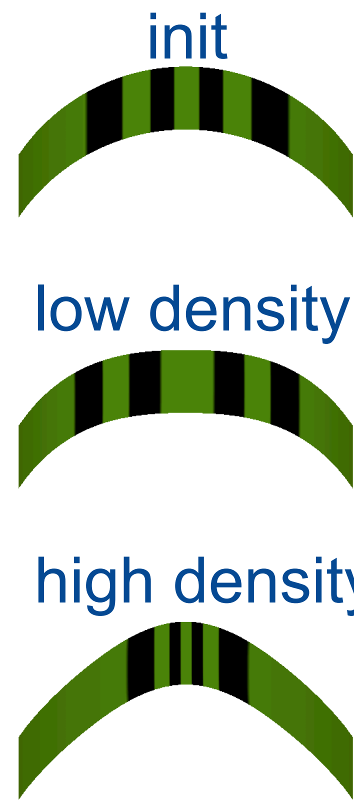


5x Gain

10x Gain

Reflection Line Manipulation

Changing density



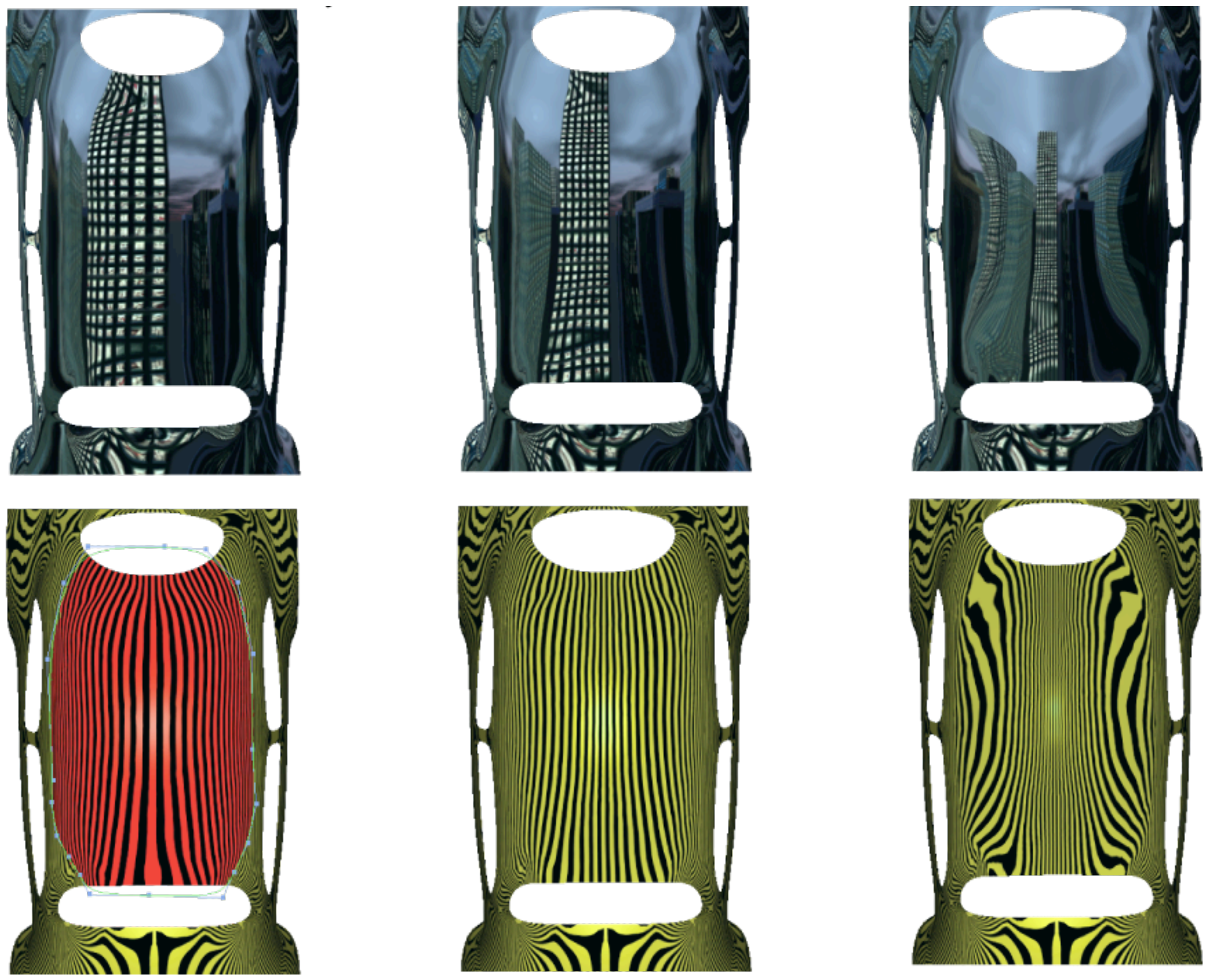
Line density Movie - WMV

Line density Movie – MP4

Reflection Line Manipulation

Changing density

low density



high density

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Reflection Line Manipulation

Changing direction

Rotation Movie - WMV
Rotation Movie – MP4

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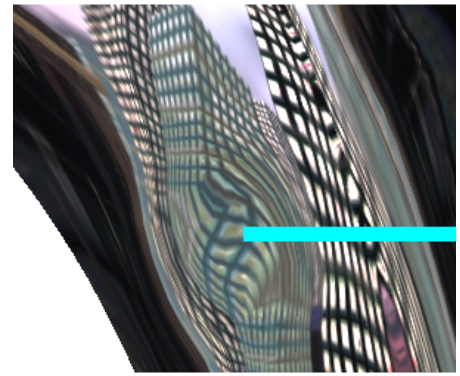
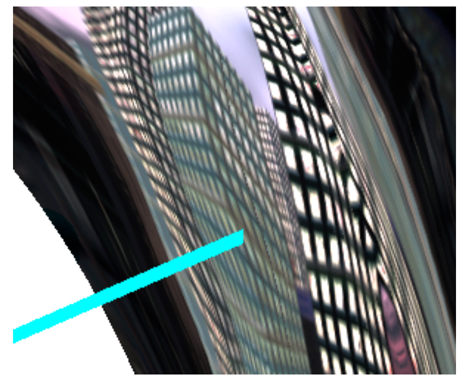
Reflection Line Manipulation

Changing direction



Car example movie - WMV
Car example movie - MP4

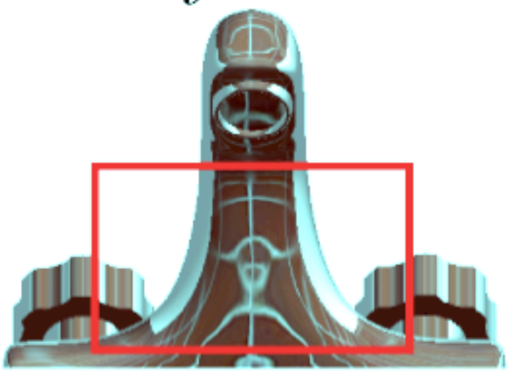


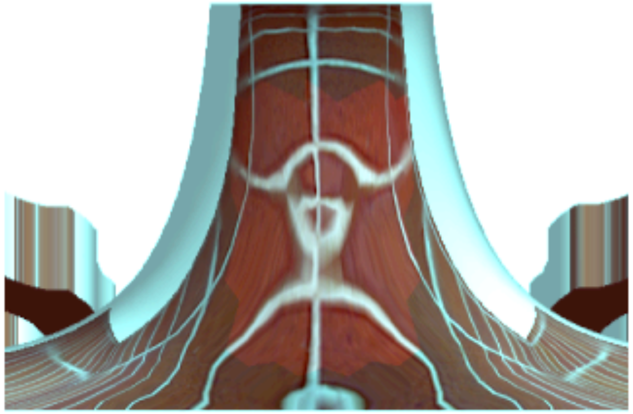
Reflection Line Manipulation

Smoothing reflection lines

- Target values through smoothing

before

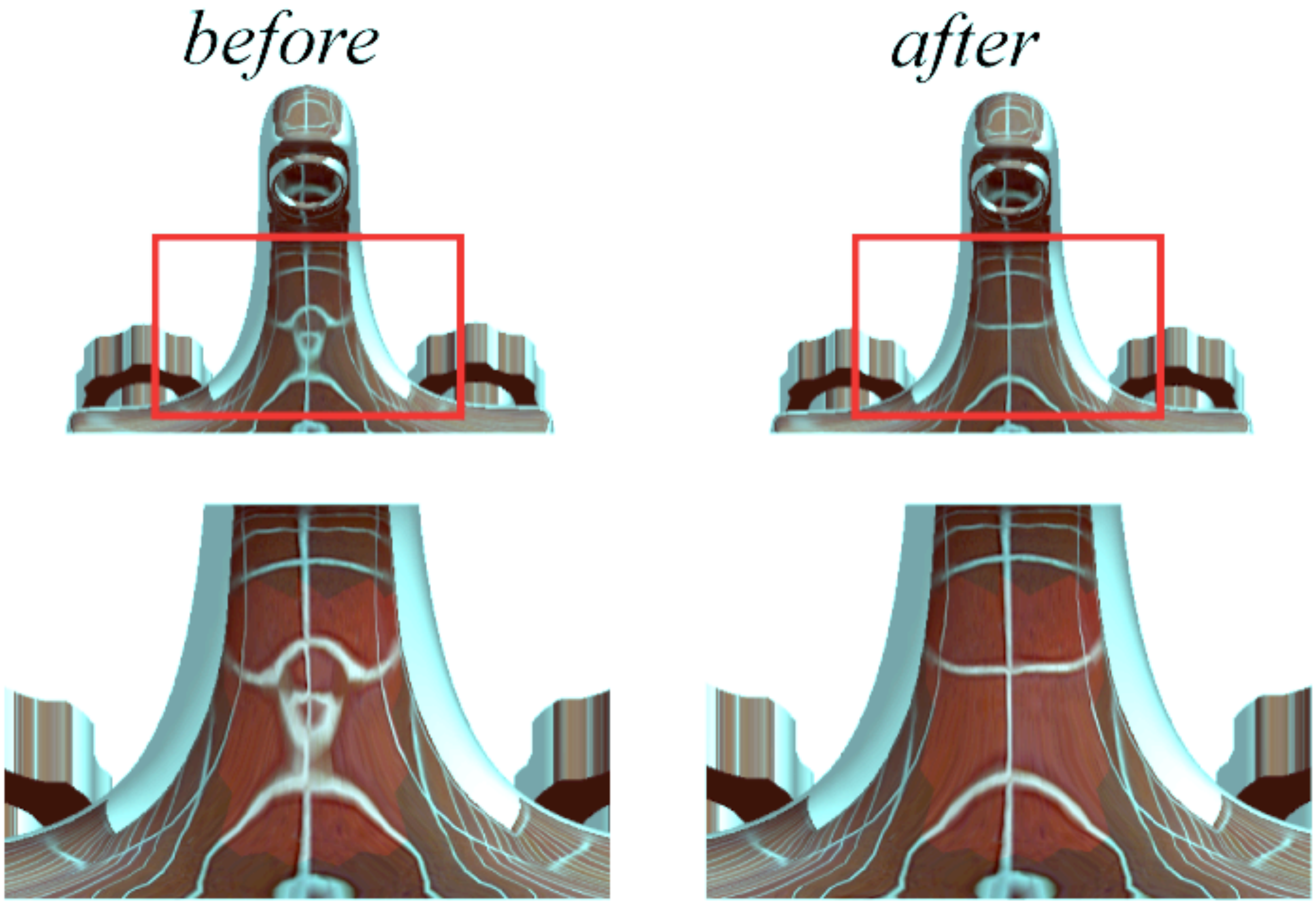




Reflection Line Manipulation

Smoothing reflection lines

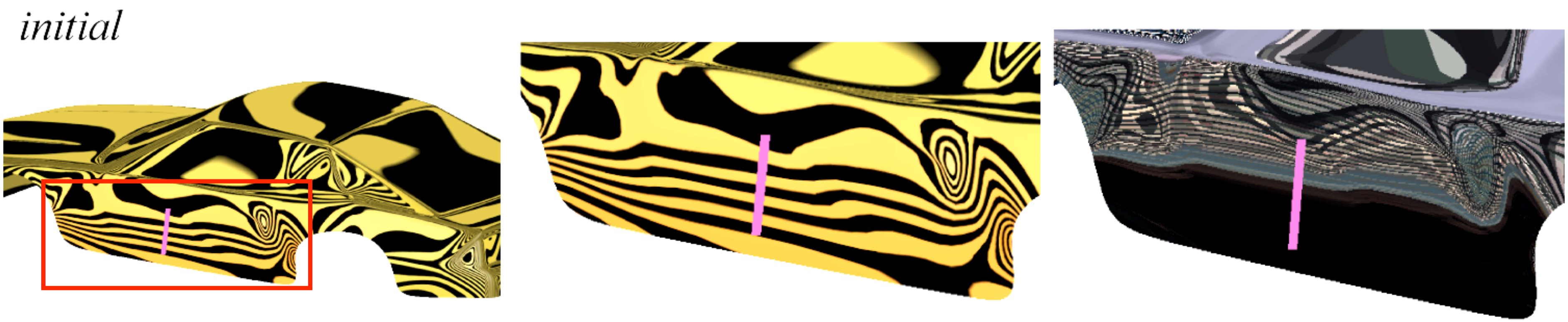
- Target values through smoothing



Reflection Line Manipulation

Smoothing reflection lines

- Target values through smoothing
- Directional smoothing



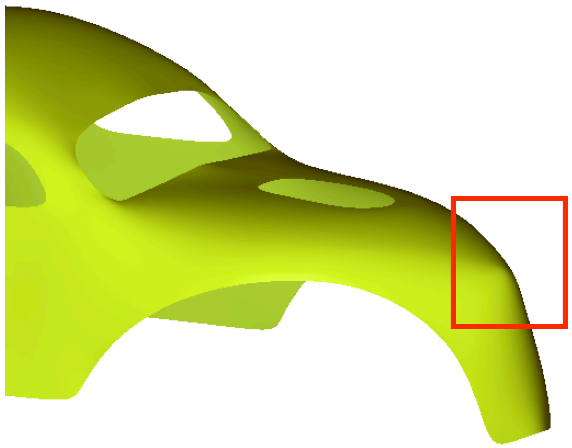
Reflection Line Manipulation

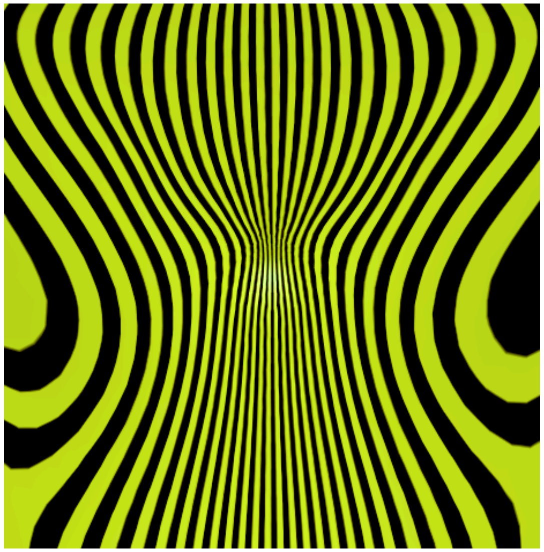
Smoothing reflection lines

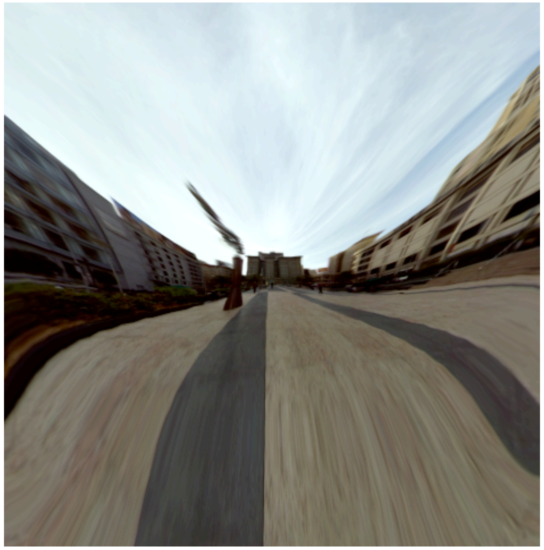
- Target values through smoothing
- Directional smoothing

Reflection Line Manipulation

Warping







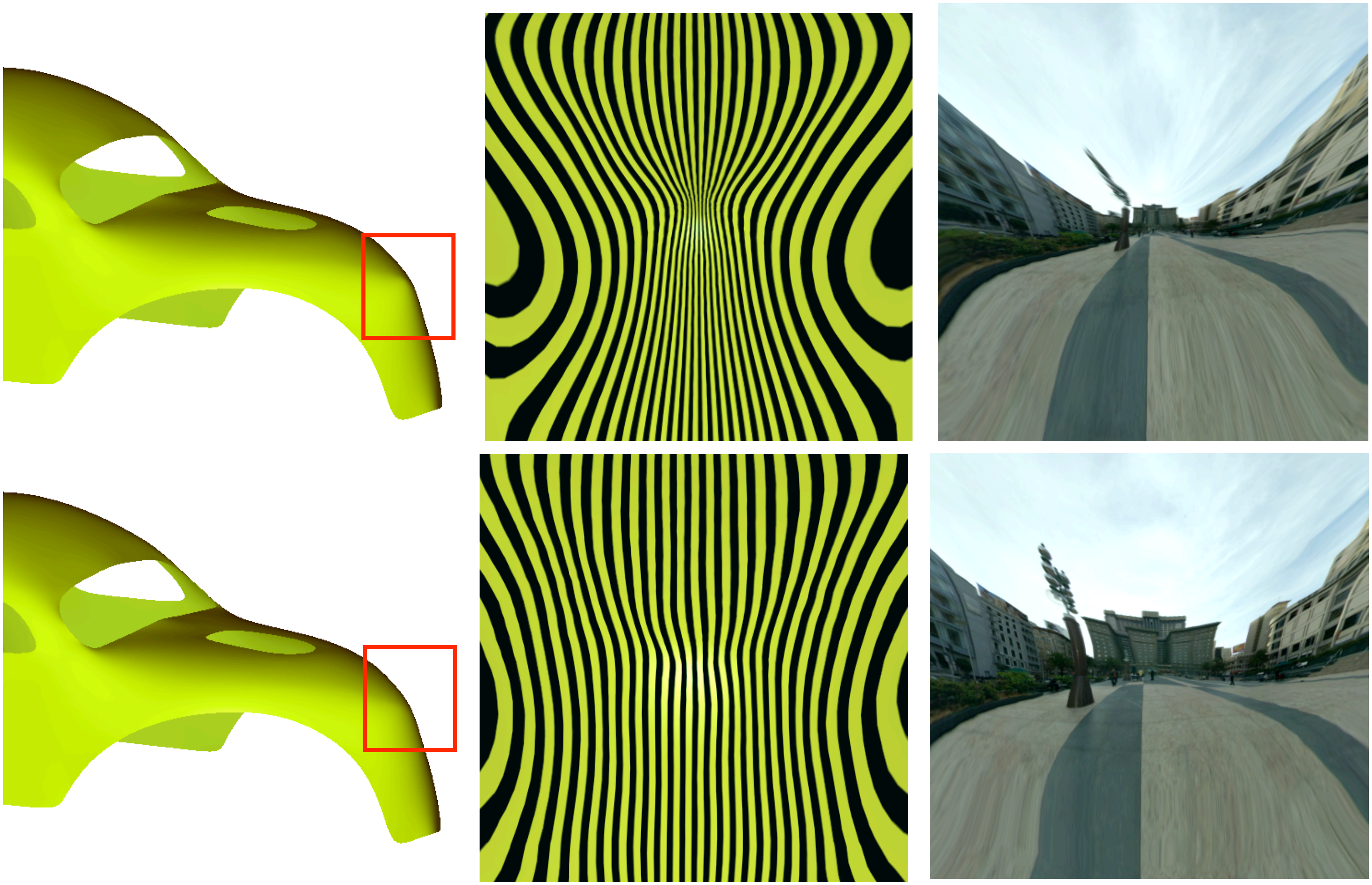
Reflection Line Manipulation
Warping

Warping on car movie - WMV

Warping on car movie – MP4

Reflection Line Manipulation

Warping



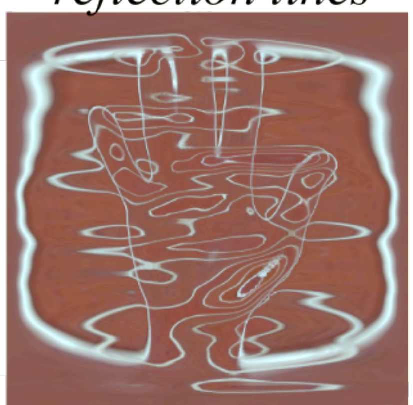
Reflection Line Manipulation

Image based reflection pattern

reflection function

reflection lines

original *blurred*



environment map

Conclusions/Future Work

Interactive system to optimize shapes of surfaces based on reflection lines

- Image-plane parameterization
- Simple triangle-based Hessian discretization

Future Work

- Integration with silhouette editing of [Nealen, Sorkine, Alexa and Cohen-Or 2005]

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